$\Delta S = 2$ decays of B^- meson in MSSM and two Higgs doublet model

S. Fajfer[a] and P. Singer[b]

[a]Physics Department, University of Ljubljana and J. Stefan Institute, 1000 Ljubljana, Slovenia

[b]Department of Physics, Technion - Israel Institute of Technology, Haifa 32000, Israel

In view of the extreme smallness of $\Delta S = 2$ transitions of B meson in the Standard Model, we consider their occurence in several extensions of it. Thus, we analyze the three - body $B^- \to K^- K^- \pi^+$ and two - body $B^- \to K^{*-} \bar{K}^{*0}$, $B^- \to K^- \bar{K}^0$, $B^- \to K^{*-} \bar{K}^{*0}$, $B^- \to K^- \bar{K}^{*0}$ decay modes both in the Standard Model and in the Minimal Supersymmetric Model with and without $\mathcal{R}$ parity conservation and in two Higgs doublet models. All five modes are found to have a branching ratio of the order of 10^{-13} in the Standard Model, while the expected branching ratio in the different extensions vary between $10^{-9} - 10^{-6}$, for a given reasonable choice of parameters.

The rare B meson decays are very important in current searches of physics beyond Standard Model (SM) [1]. Recently, it has been suggested [2–4] to investigate effects of new physics possibly arising from $b \to ss\bar{d}$ or $b \to dd\bar{s}$ decays. As shown in Ref. [2], the $b \to ss\bar{d}$ transition is mediated in the standard model by the box-diagram and its calculation results in a branching ratio of nearly 10^{-11}, the exact value depending on the relative unknown phase between t, c contributions in the box. The authors of Refs. [2,3] have calculated the $b \to ss\bar{d}$ transition in various extensions of the SM. It appears that for certain plausible values of the parameters, this decay may proceed with a branching ratio of $10^{-8} - 10^{-7}$ in the minimal supersymmetric standard model (MSSM) and in two Higgs doublet models [3].

Thus, decays related to the $b \to ss\bar{d}$ transition which was calculated to be very rare in the Standard Model, provide a good opportunity for investigating beyond the Standard Model physics. Moreover, when one considers supersymmetric models with $\mathcal{R}$-parity violating couplings, it turned out that the existing bounds on the involved couplings of the superpotential did not provide any constraint on the $b \to ss\bar{d}$ mode [2]. Recently, the OPAL collaboration [5] has set lower bounds on these couplings from the establishment of an upper limit for the $B^- \to K^- K^- \pi^+$ decay $BR(B^- \to K^- K^- \pi^+) \leq 8.8 \times 10^{-5}$. Here we investigate the branching ratio

of this decay mode in MSSM, with and without $\mathcal{R}$ parity and two Higgs doublet models as possible alternatives to the SM. Then we comment on another possibilty for the observation of the $b \to ss\bar{d}$ transition: the two body decays of B^-. First, we proceed to describe the framework used in our analysis in which we concentrate on MSSM, with and without $\mathcal{R}$ parity and two Higgs doublet models.

The minimal supersymmetric extension of the Standard Model leads to the following effective Hamiltonian describing the $b \to ss\bar{d}$ transition [2,6]

$$\mathcal{H} = \tilde{C}_{MSSM}(\bar{s}\gamma^\mu d_L)(\bar{s}\gamma_\mu b_L), \qquad (1)$$

where we have denoted

$$\tilde{C}_{MSSM} = -\frac{\alpha_s^2 \delta_{12}^{d*}\delta_{23}^d}{216 m_{\tilde{d}}^2}[24 x f_6(x) + 66\tilde{f}_6(x)]\,(2)$$

with $x = m_{\tilde{g}}^2/m_{\tilde{d}}^2$, and the functions $f_6(x)$ and $\tilde{f}_6(x)$ are given in [6]. The couplings δ_{ij}^d parametrize the mixing between the down-type left-handed squarks. At the scale of b quark mass and by taking the existing upper limits on δ_{ij}^d from [6] and [2] the coupling $\tilde{C}_{MSSM}$ is estimated to be $|\tilde{C}_{MSSM}| \leq 1.2 \times 10^{-9}\ GeV^{-2}$ for an average squark mass $m_{\tilde{d}} = 500\ GeV$ and $x = 8$, which leads to an inclusive branching ratio for $b \to ss\bar{d}$ of 2×10^{-7} [2]. The corresponding factor calculated in SM, taking numerical values from [7]

and neglecting the CKM phases is estimated to be $|C_{SM}| \simeq 4 \times 10^{-12}$ [2].

The authors of [2] have also investigated beyond MSSM cases by including R- parity violating interactions. The part of the superpotential which is relevant here is $W = \lambda'_{ijk}L_iQ_jd_k$, where i,j,k are indices for the families and L,Q,d are superfields for the lepton doublet, the quark doublet, and the down-type quark singlet, respectively. Following notations of [8] and [2] the tree level effective Hamiltonian is

$$\mathcal{H} = -\sum_n \frac{f_{QCD}}{m_{\tilde{\nu}_n}^2}[\lambda'_{n32}\lambda'^*_{n21}(\bar{s}_R b_L)(\bar{s}_L d_R)$$
$$+\lambda'_{n21}\lambda'^*_{n32}(\bar{s}_R d_L)(\bar{s}_L b_R)]. \qquad (3)$$

The QCD corrections were found to be important for this transition [9]. For our purpose it suffices to follow [2] retaining the leading order QCD result $f_{QCD} \simeq 2$, for $m_{\tilde{\nu}} = 100\ GeV$.

Most recently an upper bound on the specific combination of couplings entering (3) has been obtained by OPAL from a search for the $B^- \to K^-K^-\pi^+$ decay [5] $\sum_n(|\lambda'_{n32}\lambda'^*_{n21}|^2 + |\lambda'_{n21}\lambda'^*_{n32}|^2)^{1/2} < 10^{-4}$. Here we take the order of magnitude, while the OPAL result is 5.9×10^{-4} based on a rough estimate $\Gamma(B^- \to K^-K^-\pi^+) \simeq 1/4\ \Gamma(b \to ss\bar{d})$.

The decay $b \to ss\bar{d}$ has been investigated using two Higgs doublet models (THDM) as well [3]. These authors found that the charged Higgs box contribution in MSSM is negligible. On the other hand, THDM involving several neutral Higgses [10] could have a more sizable contribution to these modes. The part of the effective Hamiltonian relevant in our case is the tree diagram exchanging the neutral Higgs bosons h (scalar) and A (pseudoscalar) (see for details [3,15]).

$$\mathcal{H}_{TH} = \frac{i}{2}\xi_{sb}\xi_{sd}(\frac{1}{m_h^2}(\bar{s}d)(\bar{s}b)$$
$$-\frac{1}{m_A^2}(\bar{s}\gamma_5 d)(\bar{s}\gamma_5 b)), \qquad (4)$$

The coupling ξ_{ij} defined in [10] as a Yukawa coupling of the FCNC transitions $d_i \leftrightarrow d_j$. In our estimation we use the bound $|\xi_{sb}\xi_{sd}|/m_H^2 > 10^{-10}\ GeV^{-2}$, $H = h, A$, which was obtained in [3] by using the Δm_K limit on ξ_{bd}/m_H and assuming

$|\xi_{sb}/m_H| > 10^{-3}$.

We proceed now to study the effect of Hamiltonians (1), (3) on the various two body $\Delta S = 2$ decays of charged B - mesons. In order to calculate the matrix elements of the operators appearing in the effective Hamiltonian, we use the factorization approximation [11–13], which requires the knowledge of the matrix elements of the current operators or the density operators. Here we use the standard form factor representation [12,11] of the matrix elements described in detail in [4,15].

For the F_1 and F_0 form factors appearing in the decomposition of the matrix element of the weak current between two pseudoscalar states, one usually assumes pole dominance [12,14]. For the vector and axial vector form factor, appearing in the decomposition of the matrix element of the week current between the vector and pseudoscalar states, we use again pole dominance [12,14]. The relevant parametrs are taken from [11,13] $F_0^{BK}(0) = 0.38$, $A_0^{BK*}(0) = 0.32$. For the calculations of the density operators we use derivatives of the vector or axial-vector currents [15]. In our numerical calculation we use $f_K = 0.162\ GeV$, $g_{K*} = 0.196\ GeV^2$ [13]. Now we turn to the analysis of the specific modes.

We denote $\mathcal{O} = (\bar{s}\gamma^\mu(1-\gamma_5)d)\ (\bar{s}\gamma_\mu(1-\gamma_5)b)$, and then we use $\mathcal{H} = C\mathcal{O}$ with C being $1/4\tilde{C}_{MSSM}$, $1/4C_{SM}$ [15]. Using factorization and introducing $s = (p_B - k_1)^2$, $t = (p_B - k_2)^2 = $ and $u = (p_B - p_\pi)^2$, one finds for the $B^- \to K^-K^-\pi^+$ decay

$$\langle K^-(k_1)K^-(k_2)\pi^+(p_\pi)|\mathcal{O}|B^-(p_B)\rangle =$$
$$F_1^{K\pi}(s)F_1^{BK}(s)[m_B^2 + 2m_K^2 - s - 2t-$$
$$-\frac{m_K^2 - m_\pi^2}{s}(m_B^2 - m_K^2)]$$
$$+F_0^{K\pi}(s)F_0^{BK}(s)\frac{m_K^2 - m_\pi^2}{s}(m_B^2 - m_K^2). \qquad (5)$$

Within MSSM the branching ratio is found to be

$$BR(B^- \to K^-K^-\pi^+)_{MSSM} \le 4.7 \times 10^{-9}, \qquad (6)$$

while SM gives this rate to be 5.2×10^{-14}. The MSSM which includes $\mathcal{R}$ parity breaking terms can occur in this decay. The matrix element of

the operator $\mathcal{O}_\mathcal{R} = (\bar{s}(1 \pm \gamma_5)d)\ (\bar{s}(1 \mp \gamma_5)b)$ is found to be

$$\langle K^-(k_1)K^-(k_2)\pi^+(p_\pi)|\mathcal{O}_\mathcal{R}|B^-(p_B)\rangle =$$
$$F_0^{K\pi}(s)F_0^{BK}(s)\frac{(m_B^2 - m_K^2)(m_K^2 - m_\pi^2)}{(m_s - m_d)(m_b - m_s)}. \quad (7)$$

Taking the values of the quark masses as in [11] $m_b = 4.88\ GeV$, $m_s = 122\ MeV$, $m_d = 7.6\ MeV$ and using the bound given above, we estimate the upper limit of the branching ratio $BR(B^- \to K^-K^-\pi^+)_\mathcal{R} \leq 1.8 \times 10^{-7}$. This limit can be raised to 6×10^{-6} for the upper bound on the couplings of 5.9×10^{-4} given in [5].

The two Higgs doublet model, with the limit $|\xi_{sb}\xi_{sd}|/m_H^2 > 10^{-10}\ GeV^{-2}$, results in a branching ratio of the order 10^{-10}.

The long distance effects (LD) are usually suppressed in the B meson decays. However, in any search of new physics one has to include their contributions also [4]. In the case of $B^- \to K^-K^-\pi^+$ decay, we have analyzed two contributions [4]: (I) the box diagram, which is essentially the LD analog of the SD calculation in the standard model [2] of the $b \to ss\bar{d}$ transition. (II) the contribution of virtual "D^0" and "π^0" mesons, via the chain $B^- \to K^-$"D^0"("π^0") $\to K^-K^-\pi^+$. This contribution arises as a sequence of two $\Delta S = 1$ transitions and may lead to final $K^-K^-\pi^+$ state as well. It is therefore necessary to have an estimate of its relevance vis - à - vis the "direct" $\Delta S = 2$ transition. The box diagrams contributes to the real and imaginary part of the amplitude for the $B^- \to K^-K^-\pi^+$ decay. We have found that the real part, for a reasonable value of the cut-off parameter $\Lambda \simeq 10$ GeV results in the rate 8×10^{-15}, while the imaginary part of the amplitude leads to the rate 6×10^{-12}. The largest nonresonant contribution of the "D^0" or "π^0" exchange comes from the "D^0", giving a branching ratio smaller than 10^{-13}. Therefore, we have shown that the long - distance contributions to $B^- \to K^-K^-\pi^+$ in the SM are smaller or comparable to the short - distance box diagram, and have the branching ratio in a $10^{-12} - 10^{-11}$ range. This is a most welcome feature since it strengthens the suitabilty of the $B^- \to K^-K^-\pi^+$ decay as an ideal testing ground for physics beyond the standard model, as originally suggested in ref. [2].

We briefly discuss the two - body $\Delta S = 2$ decays of B^- meson. Although in principle two body decays would appear to be simpler to analyze, there is the complication of $K^0 - \bar{K}^0$ mixing. Hence one needs also a good estimate for the $b \to s\bar{s}d$ transitions as well. For the analysis of pseudoscalar meson decay to two vector mesons $B^- \to K^{*-}\bar{K}^{*0}$ it is convenient to use helicity formalism (see details in [15]). Within MSSM model the branching ratio becomes $\leq 6.2 \times 10^{-9}$, while SM gives this rate to be 6.8×10^{-14}. The $\mathcal{R}$ - parity term described by the effective Hamiltonian (4) cannot be seen in this decay mode when factorization approach is used, since the density operator matrix element $\langle\bar{K}^{*0}|(\bar{s}d)|0\rangle$ vanishes. The two Higgs doublet model also cannot be tested in this mode due to the same reason.

The use of factorization technique described above gives following results in the case of $B^- \to K^{*-}\bar{K}^0$ decay: within MSSM the branching ratio is straightforwardly found to be $BR(B^- \to K^{*-}\bar{K}^0)_{MSSM} \leq 1.6 \times 10^{-9}$ [15], which is comparable to the SM prediction of Ref. [11] for the $\Delta S = 0$ $B^- \to K^{*-}K^0$ decay given as $BR(B^- \to K^{*-}K^0) = 1\times10^{-9}, 5\times10^{-9}, 2\times10^{-9}$ obtained for the number of colours $N_c = 2$, $N_c = 3$, $N_c = \infty$, respectively. The SM calculation for the $\Delta S = 2$ transition leads to $BR(B^- \to K^{*-}\bar{K}^0)_{SM} = 1.7 \times 10^{-14}$. The MSSM which includes $\mathcal{R}$ parity breaking terms can occur in this decay. The estimation of the upper limit of the branching ratio gives $BR(B^- \to K^{*-}\bar{K}^0)_\mathcal{R} \leq 4.4 \times 10^{-8}$. This limit can be raised to 1.5×10^{-6} for the upper bound on the couplings of 5.9×10^{-4} given in [5]. The two Higgs doublet model (4) gives for the limit $|\xi_{sb}\xi_{sd}|/m_H^2 > 10^{-10}\ GeV^{-2}$, a branching ratio of the order 10^{-11}. Due to specific combination of the products of the scalar (pseudoscalar) densities this is the only decay which has nonvanishing amplitude within the factorization assumption.

For the $B^- \to K^-\bar{K}^{*0}$ decay mode the branching ratio in MSSM is constrained to be $BR(B^- \to K^-\bar{K}^{*0})_{MSSM} \leq 5.9 \times 10^{-9}$ in comparison with SM result 6.5×10^{-14}. The amplitude calculated

in MSSM including $\mathcal{R}$ breaking and THDM vanishes, due to vanishing of the matrix element of the density operator for $\bar{K}^{*0}$ state.

The $B^- \to K^- \bar{K}^0$ decay offers the following: the branching ratio for MSSM is found to be $BR(B^- \to K^- \bar{K}^0)_{MSSM} \leq 2.3 \times 10^{-9}$, in comparison with the 2.5×10^{-14} found in the SM. The matrix element of the R parity breaking MSSM operator has nonvanishing value and the constraint on the coupling constants $\leq 10^{-4}$ gives the bound 9.4×10^{-8}, while for the bound of 5.9×10^{-4} for the coupling constants the rate $BR(B^- \to K^- \bar{K}^0)_{\mathcal{R}}$ can reach 3.3×10^{-6}.

One might wonder if the long distance effects are important in two - body $\Delta S = 2$ B^- decays. We have estimated the tree level contribution of the $D(D^*)$ which then goes into $K(K^*)$ via weak annihilation. We found that these contributions give a branching ratio of the order 10^{-18} and therefore they can be safely neglected. One might think that the exchange of two intermediate states $D(D^*)$, $K(K^*)$ can introduce certain long distance contributions. In decay $B \to "D" "K" \to "K" "K"$ the first weak vertex arises from the decay $B \to "D" "K"$ and the second weak vertex (see e.g. [4]) can be generally obtained from the three body decays of $D \to KKK$. Therefore, we are quite confident to suggest that the long distance effects are not important in the two - body $\Delta S = 2$ B decays.

We can summarize that in the $B^- \to K^- K^- \pi^+$ and two - body B^- decays, the MSSM with the chosen set of parameters gives rates of the order $10^{-9} - 10^{-8}$, while the $\mathcal{R}$ parity breaking terms in the MSSM can be seen only in the $B^- \to K^- K^- \pi^+$, $B^- \to K^{*-} \bar{K}^0$ and $B^- \to K^- \bar{K}^0$ decays. Let us turn now to the possibility of detecting these decay modes. The $B^- \to K^- K^- \pi^+$ seems to be the best candidate, since the other modes we discussed, have a $\bar{K}^0$ in the final states which complicates the possibilty of a detection because of $K^0 - \bar{K}^0$ mixing [15]. These are the modes which as we mentioned are more difficult on the experimental side. The THDM model can give nonvanishing contribution in the case of $B^- \to K^- K^- \pi^+$ and $B^- \to K^{*-} \bar{K}^0$ decays, with a rate too small to be seen. Thus, we con-

clude that the $B^- \to K^- K^- \pi^+$ decay is an ideal candidate to look for physics beyond SM and that the $B^- \to K^{*-} \bar{K}^{*0}$, $B^- \to K^- \bar{K}^{*0}$ decays offer this possibility also.

REFERENCES

1. For reviews, see A. Masiero, L. Silvestrini in Proc. 2nd Intern. Conf. on B Physics and CP Violation (BCONF 97), Honolulu, HI 1997, T. E. Browder, F. A. Harris, S. Pakvasa eds (World Scientific, Singapore 1998), p.172. Y. Grossman, Y. Nir, R. Rattazzi in "Heavy Flavours" (2nd Edition), A. J. Buras and M. Lindner eds, p. 755 (World Scientific, Singapore 1998).
2. K. Huitu, C. -D.Lü , P. Singer, D. -X. Zhang, Phys. Rev. Lett. 81 (1998) 4313.
3. K. Huitu, C. -D.Lü , P. Singer, D. -X. Zhang,Phys. Lett. B 445 (1999) 394.
4. S. Fajfer and P. Singer, Phys. Lett. B 478 (2000) 185.
5. G. Abbiendi et al., OPAL Collaboration, Phys. Lett. B 476 (2000) 233.
6. F. Gabbiani, E. Gabrielli, A. Masiero, L. Silvestrini, Nucl. Phys. B 477 (1996) 321.
7. Particle Data Group, D. E. Groom et. al, Europ. Phys. Journal C 15 (2000) 1.
8. D. Choudhury and P. Roy, Phys. Lett. B 378 (1996) 153.
9. J. L. A. Bagger, K.T. Matchev, and R.J. Zhang, Phys. Lett. B 412 (1997) 77.
10. T. P. Cheng, M. Sher, Phys. Rev. D 35 (1987) 3484; M. Sher, Y. Yuan, Phys. Rev. D 41 (1991) 1461.
11. A. Ali, G. Kramer and C. D. Lü, Phys. Rev. D 58 (1998) 094009.
12. M. Wirbel, B. Stech and M. Bauer, Z. Phys. C 29 (1985) 637.
13. M. Bauer, B. Stech, M. Wirbel, Z. Phys. C 34 (1987) 103; F. Buccella, M. Forte, G. Miele, G. Ricciardi, Z. Phys. C 48 (1990) 47; P. Lichard, Phys. Rev. D 55 (1997) 5385.
14. S. Fajfer, J. Zupan, Int. J. Mod. Phys. A 14 (1999) 4161.
15. S. Fajfer and P. Singer, hep-ph/0007132.